6.1 Slope Fields and Euler's Method

Solving equations involving derivatives is a branch of calculus called differential equations – very fun, interesting and EASY!!! The highest order of the derivatives involved is the order of the equation.

Ex. 1 Find all functions that satisfy

$$\frac{dy}{dx} = -6x^2 - \sec x \tan x + 3^x \ln 3$$

The family of functions is the general solution to the differential equation.

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Initial Value Problems

Ex. 2 Find the particular solution to the equation

$\frac{dy}{dx} = 3x^2 + 2x + 1$ whose graph passes through the point (1,0).

Discontinuity

Ex. 3 Find the particular solution to the equation

$$\frac{dy}{dt} = 5\sec^2 x - \frac{3}{2}\sqrt{x}$$

whose graph passes through the point (0,7).

Ex. 4 Find the solution to the differential equation

$$f'(t) = e^{-t^2}$$ for which $f(6)=5$

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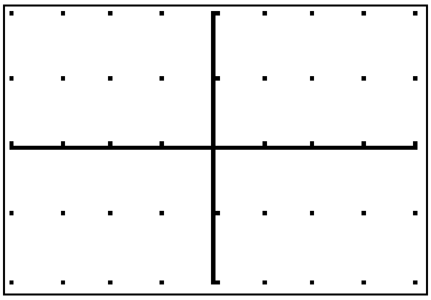
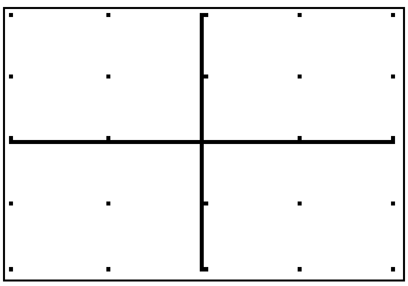
Slope Fields

Essentially we create a slope field by using local linearization at every point, constructing the graph by adding each piece together.

Ex. 5 Construct a slope field for the differential equation $\frac{dy}{dx} = \sin x$



Ex. 6 Construct a slope field for the differential equation $\frac{dy}{dx} = x + 2y$

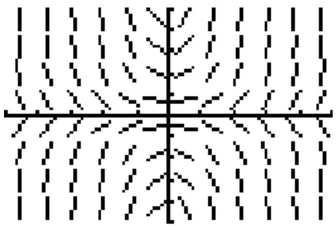


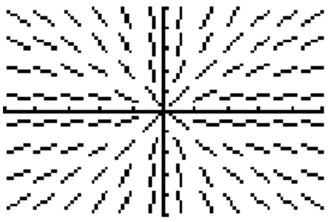
Quadrant Analysis

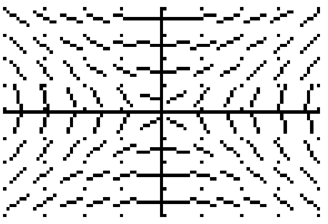
Sometimes it helps to look at how the function behaves in each quadrant when analyzing a given slope field pattern.

Ex. 7

Match the slope field for $\frac{dy}{dx} = \frac{y}{x}$ $\frac{dy}{dx} = \frac{x}{y}$ $\frac{dy}{dx} = xy$







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Euler's Method

Euler's method is a numerical method for solving differential equations. Essentially we are using local linearizations to determine our approximation.

Ex.8

Use Euler's method to approximate for f(0.6) given the initial conditions f(0)=1 for

$$\frac{dy}{dx} = y + x^2$$ use a step size Δx=0.2.

The approximation in Ex.8 is an underestimate - this happens when the curve is concave up. An overestimate is encountered when the curve is concave down.

Ex.9

If $\frac{dy}{dx} = 2x - y$ and if y=3 when x=2, use Euler's method with three equal steps to approximate y when x=1.7.